arXiv:hep-ph/9309226v1 4 Sep 1993

HIGGS SECTOR MOTIVATIONS FOR AN e^-e^- LINEAR COLLIDER[⋆]

J.F. Gunion

Davis Institute for High Energy Physics, Dept. of Physics, U.C. Davis, Davis, CA 95616

Abstract

I briefly review the crucial role an e^-e^- linear collider could play in unravelling
the nature of a non-minimal Higgs sector and/or strongly-interacting WW sector..

1. Introduction

Much has been written[1] about the virtues of a linear e^+e^- collider for probing
the Higgs sector or, more generally, revealing the nature and source of electroweak
symmetry breaking (EWSB). While an e^+e^- collider is fully adequate as a probe
of Higgs physics in the case of the minimal Standard Model (MSM), it generally
has important limitations if the Higgs sector is non-minimal or if the W ($W \equiv$
$W^{\pm}, Z$) boson sector is strongly interacting. For a perturbative non-minimal Higgs
sector, e^-e^- collisions could be crucial for observing doubly-charged Higgs bosons
via $e^-e^- \to \nu\nu W^- W^- \to \nu\nu H^{--}$ as well as various exotic couplings of neutral and
singly-charged Higgs bosons. If the W boson sector is strongly interacting, only the
combination of e^+e^- and e^-e^- collisions will allow a full investigation of all WW
scattering channels, as required to fully understand electroweak symmetry breaking.
Here I review the basic theoretical ideas and phenomenology that provide significant
motivation for retaining an e^-e^- collision option at the next linear collider (NLC).

2. Perturbative Higgs Sector Extensions

In the context of perturbative theories containing elementary Higgs bosons, the
MSM need not be nature's choice. Many generalizations have been discussed.[2] Here, I
focus entirely on extensions of the Higgs sector only. Supersymmetric generalizations
of the models to be discussed are certainly possible, but will not be considered here.

There are two crucial conditions that must be satisfied by any perturbative Higgs
sector generalization:

---

1. no problems with unitarity at high energy;

2. no violation of $\rho \simeq 1$.

Turning first to ρ, we recall the tree-level result:

$$\rho = \frac{\sum_{T,Y} \left[4T(T+1) - Y^2\right] \left|V_{T,Y}\right|^2 c_{T,Y}}{\sum_{T,Y} 2Y^2 \left|V_{T,Y}\right|^2} \tag{1}$$

where $\langle \phi(T,Y) \rangle = V_{T,Y}$ defines the vacuum expectation value of the neutral Higgs field member of the Higgs representation with total $SU(2)_L$ isospin and hypercharge specified by T and Y, respectively, and $c_{T,Y}$ is 1 for a complex representation and $1/2$ for a $Y = 0$ real representation. For a single representation to yield $\rho = 1$ we require $(2T+1)^2 - 3Y^2 = 1$, which is satisfied for any number of Higgs doublets ($T = 1/2$, $Y = \pm 1$), to which any number Higgs singlets ($T = Y = 0$) can be added.

The MSM employs a single Higgs doublet. The most attractive extension is to two Higgs doublet fields. This extension, including its supersymmetric generalization is reviewed at length in Ref. 1. In general, $e^- e^-$ colliders are not particularly critical to fully explore such a generalization. However, higher representations should certainly be given consideration. The next possible single-representation solution to Eq. (1) is $T = 3$, $Y = 4$. Normally it is discarded because of its complexity. The simplest representation beyond the doublet is a Higgs triplet, $T = 1$. In order to have a neutral member the only possible hypercharge values are $Y = 0, \pm 2$. As we see from Eq. (1), $\rho = 1$ is not automatic in such a case. Various possibilities for obtaining $\rho \simeq 1$ at tree level can be entertained. First, it could be that a single triplet occurs in combination with a doublet. However, since current experimental limitations on $\Delta\rho$ are in the vicinity of 0.002 (the exact number depends upon the confidence level criterion), the vacuum expectation values of the neutral triplet field must be quite small, implying that the triplets would play little role in EWSB. For example, including one or the other of the two triplet representations cited above, one finds that $|V_{1,0}|/|V_{1/2,1}| \sim 0.03$, $|V_{1,\pm 2}|/|V_{1/2,1}| \sim 0.03$ yields $\rho \simeq 1.002$, $\rho \simeq .998$, respectively. (Note that in the first case $\Delta\rho > 0$, while in the second $\Delta\rho < 0$.) Of course, it will be important to determine if a triplet representation is present, regardless of the vacuum expectation value of its neutral member. But, as we shall see, for small vacuum expectation value, an $e^- e^-$ collider would not be useful for probing the triplet.

Thus, let us consider scenarios in which the triplet vacuum expectation value could be large. The first such possibility arises if m_t is very large. In this case the $t-b$ doublet yields a large positive contribution to $\Delta\rho$ which could by cancelled by the negative $\Delta\rho$ that would arise from a $T = 1$, $Y = \pm 2$ complex triplet representation. Obviously, this would require fine tuning the triplet vacuum expectation value to achieve $\rho \simeq 1$. In such a scenario, quite large values of m_t are required for $V_{1,\pm 2}$ to be big enough to allow the triplet to be an important player in electroweak symmetry

breaking. For example, to compensate for a ratio of $|V_{1,\pm 2}|/|V_{1/2,1}| \sim 0.2$ we would need $m_t \sim 475$ GeV.

A third, and in my opinion the most attractive, possibility is to combine one doublet Higgs field with one real ($T = 1, Y = 0$) and one complex ($T = 1, Y = 2$) field. (For a review and references, see Ref. 2.) If the Higgs potential is adjusted so that it has a custodial $SU(2)$ symmetry at tree-level, then the neutral members of the two triplet representations have the same vacuum expectation value and, as can be explicitly verified using Eq. (1), $\rho = 1$ at tree-level. Denoting $V_{1/2,1} = a/\sqrt{2}$ and $V_{1,0} = V_{1,2} = b$, we find $m_W^2 = \frac{1}{4}g^2 v^2$ with $v^2 \equiv a^2 + 8b^2$. The importance of the triplet fields in electroweak symmetry breaking can be characterized by $\tan\theta_H \equiv 2\sqrt{2}b/a$. There is nothing to prevent having $b >> a$, in which case EWSB would be dominated by the triplet fields. We will shortly return to the phenomenology of this model and the important role an $e^- e^-$ collider would play.

However, I should first explain why this one-doublet, two-triplet model is not as attractive as a pure doublet model, despite automatically yielding $\rho = 1$ at tree-level. The difficulty is an important new fine tuning problem that arises at one-loop. It is easily verified that the hypercharge gauge interactions ($igB(Y/2)$) violate the custodial $SU(2)$ symmetry. This implies that the special form of the Higgs potential required for $V_{1,0} = V_{1,2} = b$ is *infinitely* renormalized. Thus, the coefficients in the Higgs potential must be fine-tuned to preserve $\rho = 1$ at 1-loop. Details regarding this fine-tuning appear in Ref. 3. In other words, the most general triplet potential contains $SU(2)_{custodial}$-violating terms, which, even if set equal to zero at tree-level, will be generated at one-loop. This is in contrast to the potential for a model with only Higgs doublets (plus possible singlets), for which it turns out that even the most general Higgs potential does not contain terms that can violate the custodial symmetry. For renormalizable theories (which these models are), this implies that all radiative corrections to ρ must be finite if only doublets and singlets are present, a very attractive conclusion given the experimental constraints on ρ.

Thus, triplet models are generally not in favor with theorists. However, this does not mean that such models could not be nature's choice. A complete experimental program should provide for the ability to search for the many new signatures that would arise if triplet Higgs representations are present. The phenomenology of the one-doublet, two-triplet model described above is particularly rich.[4]

At an $e^+ e^-$ collider, the most spectacular and characteristic signal for a triplet model would be the detection of the doubly charged Higgs boson(s) contained in complex Higgs triplet representation(s). But, the only available production reaction would be $e^+ e^- \to H^{++} H^{--}$. Even if adequate machine energy ($\sqrt{s} \gtrsim 2m_{H^{++}}$) is available and the H^{++} can be seen by this means, the $ZH^{++}H^{--}$ and $\gamma H^{++}H^{--}$ couplings involved in the production mechanism are fixed purely by weak-isospin and charge, and give no hint of whether the triplet Higgs field(s) play any role in EWSB. If the H^{++} decays to a (real or virtual) $W^+ W^+$ final state, the presence of this mode would indicate a non-zero value for the vacuum expectation value of the

neutral member of the associated triplet. But, the likely absence or small branching ratio of other channels would mean that the magnitude of this vacuum expectation value would be essentially impossible to extract. In order to probe the possible role of a triplet field in EWSB, other processes must be considered.

To set the stage, let us consider the coupling constant sum rules that must be satisfied in order that the theory be unitary at high energy.[5] The two most basic ones are:

$$g^2(4m_W^2 - 3m_Z^2 c_W^2) \stackrel{\rho \simeq 1}{\simeq} g^2 m_W^2 = \sum_k g_{W^+W^-H_k^0}^2 - \sum_l g_{W^+W^+H_l^{--}}^2 \,, \tag{2}$$

and

$$\frac{g^2 m_Z^4 c_W^2}{m_W^2} \stackrel{\rho \simeq 1}{\simeq} g^2 m_Z^2 = \sum_k g_{W^+W^-H_k^0} g_{ZZH_k^0} - \sum_l g_{W^+ZH_l^-}^2 \,. \tag{3}$$

(Here, $c_W \equiv \cos\theta_W$.) From these two equations we see that even if there exists a MSM-like H_1^0 such that $g_{W^+W^-H_1^0} = gm_W$ and $g_{ZZH_1^0} = gm_Z/c_W$, there is still room for more H_k^0's with (1) big $g_{W^+W^-H_k^0}$, (2) big $g_{ZZH_k^0}$ *provided*: (1) an H_l^{--} exists with $g_{W^+W^+H_l^{--}} \neq 0$; (2) $g_{W^+ZH_k^-} \neq 0$ for some singly charged Higgs. Determination that one of these couplings is non-zero would show absolutely that triplet Higgs representations exist *and* that they play an important role in EWSB. For instance, in the one-doublet, one-real-triplet, one-complex-triplet model described above there is a five-plet (under $SU(2)_{custodial}$) of Higgs bosons with degenerate masses (H_5^{--}, H_5^-, H_5^0, H_5^+, H_5^{++}) and WW-couplings specified below:

$$g_{H_5^+W^-Z} = -gm_W s_H/c_W \,, \quad g_{H_5^{++}W^-W^-} = \sqrt{2}gm_W s_H \,,$$
$$g_{H_5^0W^+W^-} = -\tfrac{1}{2}g_{H_5^0ZZ}c_W^2 = gm_W s_H/\sqrt{3} \,, \tag{4}$$

where we have defined $s_H \equiv \sin\theta_H$. If the triplets are important in EWSB, then s_H is substantial and the couplings in question are of the same order as that of the MSM Higgs to W^+W^-, ZZ.

How can we look for such couplings? The most direct technique is to look for a production process that can occur only if a given coupling is present. At an e^+e^- collider $g_{H_5^+W^-Z} \neq 0$ leads to $e^+e^- \to e^+\nu H_5^-$ via W^-Z fusion. Despite the kinematically favorable fact that only a single H_5 must be produced, the Z couples weakly to the electron, and so this process does not have a very high rate in practice. A much more dramatic demonstration of the triplet Higgs bosons' role in EWSB over a larger m_{H_5} mass region would be possible at an e^-e^- collider via observation of $e^-e^- \to \nu\nu H_5^{--}$, occurring by W^-W^- fusion. This has a rate that is fully competitive with that normally associated with W^-W^+ fusion to a MSM Higgs boson.[4]

For instance, in units of the standard R, one finds a production rate of $R \gtrsim 0.1$ for $\sqrt{s} \gtrsim 300 + m_{H_5}$ if $\sqrt{2} s_H \sim 1$. This means that a $\sqrt{s} = 500$ GeV $e^- e^-$ collider (with luminosity comparable to that normally assumed for an $e^+ e^-$ collider of this energy) could easily observe this reaction for $m_{H_5^0}$ up to about $200 - 250$ GeV. While the mass reach is only slightly better than is achievable via $e^+ e^- \to H_5^{--} H_5^{++}$, the $W^- W^-$ fusion reaction probes the $W^- W^- H_5^{++}$ coupling that is crucial to the role of the five-plet in electroweak symmetry breaking.

As an aside, let us imagine the following 'frustrating' situation that might arise if only $e^+ e^-$ collisions are available. In the one-doublet, two-triplet model being considered, there are altogether three neutral Higgs bosons with coupling to $W^+ W^-$. As well as the H_5^0, we have the H_1^0 and $H_1^{0\prime}$. If $\tan \theta_H$ is large (*i.e.* the triplets dominate EWSB), then the H_1^0 (which for small $\tan \theta_H$ plays the role of the MSM Higgs boson) has small coupling to $W^+ W^-$ and in $e^+ e^-$ collisions one would see only the H_5^0 and $H_1^{0\prime}$ in $W^+ W^-$ fusion processes. The former has $g_{H_5^0 W^+ W^-}^2 = \frac{1}{3} g_{H_{MSM} W^+ W^-}^2$, and the latter $g_{H_1^{0\prime} W^+ W^-}^2 = \frac{8}{3} g_{H_{MSM} W^+ W^-}^2$, where $g_{H_{MSM} W^+ W^-} = g m_W$ denotes the coupling strength of the MSM Higgs boson to $W^+ W^-$. Quite possibly (depending on the masses of the H_5^0 and $H_1^{0\prime}$) only the latter would yield a visible rate. One would observe a neutral Higgs boson, but with a production cross section much larger than expected in the Standard Model. Without the $e^- e^-$ collision option, it would be impossible to do more than guess at the full Higgs sector structure.

Finally, let me briefly review the decays of the doubly-charged Higgs bosons of the model. If s_H is not small, the two-body decay $H_5^{--} \to W^- W^-$ would almost certainly be dominant if kinematically allowed. Other possible two-body final states are $H_3^- W^-$ and $H_3^- H_3^-$ (where H_3^- is a member of a surviving $SU(2)_{custodial}$-triplet Higgs species in the model).[⋆] If s_H is very small, the H_3 modes might dominate if kinematically allowed. If all these two-body decays are forbidden, then three-body decays $H_5^{--} \to W^- W^{-*}$ (where $W^{-*} \to \ell^- \nu$, *e.g.*) and/or $H_3^- W^{-*}$ would be dominant.[†] If no three-body decays are kinematically accessible, then the $W^{-*} W^{-*}$ four-body decay would be the mode of choice unless s_H were extremely small. Of course, we should not forget the possibility of a $H_5^{--} \to \ell^- \ell^-$ coupling. If present at a reasonable level, the consequent decay would be dominant unless the two-body channels were kinematically allowed.[‡] This summary makes more explicit the point noted earlier. Because of the exotic charge of the H_5^{--}, it can decay to only a very

---

⋆ The H_3^- decays primarily to the heaviest allowed fermion–anti-fermion pair channel.

† The virtual H_3^- option is small due to the $H_3^- \to f \overline{f}$ coupling being proportional to m_f / m_W and, hence, small relative to the $W^- \to f \overline{f}$ coupling.

‡ Note, however, that it is unlikely that this coupling could be large enough to yield direct H_5^{--} production in $e^- e^-$ collisions at an observable level. Nonetheless, an observable rate cannot be ruled out altogether, and such production should be looked for. It would provide a very dramatic and unique use for $e^- e^-$ collisions.

few channels, each of which has a highly model-dependent strength. Thus, it would be difficult to quantitatively determine the role of the triplet Higgs fields in EWSB simply by examining the H_5^{--} decays.

The only significant background to detection of the H_5^{--} in the W^-W^- *etc.* final states would derive from the irreducibly present $W^-W^- \to W^-W^-$ electroweak subprocesses (*i.e.* those present whether or not there is a Higgs boson). For a light H_5^{--} resonance, this background would present no problem. If the H_5^{--} is heavy, a more general discussion of W^-W^- scattering is appropriate, and is my next topic.

3. Strong $\mathbf{W_L^- W_L^-}$ Scattering

There is no guarantee that the EWSB sector will be entirely perturbative, or even partially perturbative. For instance, if there are no Higgs bosons then we must consider the possibility that WW scattering becomes strong in all channels and that a perturbative approach is not possible. Perhaps a technicolor model will prove to be correct, perhaps some other approach. In any case, at best a very incomplete picture of WW interactions will be possible if only e^+e^- collisions and not e^-e^- collisions are available. In general, a full understanding of EWSB will emerge only if the means by which unitarity is restored at high energy can be explored in all WW channels, including the W^-W^- channel.

The signal for a strongly interacting WW sector is non-unitary growth of the $W_LW_L \to W_LW_L$ scattering amplitudes for longitudinally polarized gauge bosons at high energy. Higgs bosons are required to cancel the bad high energy growth that occurs if only gauge-boson-exchange electroweak graphs are present. The single Higgs boson of the MSM is sufficient to achieve this cancellation in all WW scattering channels. If the Higgs boson is not present or is very heavy, then every W_LW_L scattering channel will become strongly interacting. But this is only the simplest of many possibilities. To illustrate the possible complexity of unitarity cancellations, and the importance of the W^-W^- channel in being able to fully explore a scenario in which WW scattering becomes strong, we return again to the one-doublet, two-triplet model considered earlier. Aside from the purely electroweak graphs, numerous Higgs exchange graphs contribute to $W_L^-W_L^- \to W_L^-W_L^-$. These can be divided into:

1. exchanges of H_1^0, $H_1^{0\prime}$ and H_5^0 in the t-channel and u-channel; and

2. exchange of H_5^{--} in the s-channel.

Relative to MSM strength $g^2 m_W^2$, the 'effective' (*i.e.* after taking into account the signs of the high energy amplitudes) strengths of these respective graphs are c_H^2, $\frac{8}{3}s_H^2$, $\frac{1}{3}s_H^2$, and $-2s_H^2$. As required, these strengths sum to the MSM result:

$$\sum = g^2 m_W^2 \left[c_H^2 + \left(\tfrac{8}{3} + \tfrac{1}{3} - 2 \right) s_H^2 \right] = g^2 m_W^2.$$

But, of course, some of the Higgs might be light, and some substantially heavier,

and the manner in which unitarity cancellations occur could be quite complex. I will illustrate using two extreme possibilities.

First, suppose $\tan\theta_H \sim 1$, so that $c_H \sim s_H$, and that H_1^0 is light whereas the other Higgs bosons are heavy. t- and u-channel exchanges of the H_1^0, with positive weight (the same sign of weight as for the single Higgs of the MSM) of $c_H^2 g^2 m_W^2 \sim \frac{1}{2} g^2 m_W^2$, would partially accomplish the required unitarity cancellation at a low energy, leaving behind a unitarity violating high energy behavior that would be weaker than that found in the MSM when its single Higgs boson is heavy. The bad high energy behavior in such a case would not need to be cured until $\sqrt{s}$ is substantially above the canonical $\sqrt{s} \sim 1.8$ TeV limit found in the MSM single-Higgs scenario.

In another extreme, suppose the (degenerate) H_5 Higgs bosons are light, and the others substantially heavier. In net, the H_5 contributions to $W_L^- W_L^-$ scattering have a weight of $-\frac{5}{3} g^2 m_W^2$, *i.e.* large and opposite in sign to the MSM Higgs. This would imply a very rapid high energy growth of the $W_L^- W_L^-$ scattering amplitude until the other Higgs exchanges entered. In a 'hybrid' model, it could happen that there are light H_5 Higgs bosons as described by the model being considered, but that the other Higgs bosons are not even present. Then, new physics would have to enter at very low energies in order to cure the unitarity-violating high energy growth in the $W_L^- W_L^-$ scattering channel deriving from electroweak and H_5-exchange graphs.

Of course, correlated phenomena would be taking place in the $W^+ W^-$ and $W^\pm Z$ channels. But, because of the multiplicity of the different Higgs bosons that might or might not be heavy (especially if in some more arbitrary scenario the degeneracy of the H_5's is broken) the $W^- W^-$ channel would absolutely be needed in order to fully understand the physics of the EWSB sector.

Certainly, high $\sqrt{s}$ for the NLC will be essential to explore EWSB if WW scattering unitary behavior is only fully achieved at WW energies in the TeV range. This is the arena for what would probably be a second generation linear collider with $\sqrt{s}$ in the $2-4$ TeV range. If a light MSM-like Higgs boson is not found at a $\sqrt{s} \lesssim 1$ TeV $e^+ e^-$ collider, then a strong WW scattering scenario becomes likely. The above illustrations show that a full understanding of such a sector will almost certainly require the ability to study $W^- W^-$ scattering as well as $W^+ W^-$, $W^+ Z$ and $W^- Z$ scattering. While the latter three processes are accessible in $e^+ e^-$ collisions, only $e^- e^-$ collisions allow a study of the first. Thus, if one arrives at a juncture where a multi-TeV linear collider is being considered, and the nature of EWSB has not been fully resolved at a lower energy machine, it is especially crucial that an $e^- e^-$ collision option be included in the machine design.

4. Conclusions

If the MSM or the minimal supersymmetric model (MSSM) is correct, then only doublet or lower Higgs representations occur and, in addition, light Higgs boson(s) should be regarded as likely. Any such light Higgs is almost certain to be discoverable

at an e^+e^- machine with $\sqrt{s} \lesssim 500$ GeV $- 1$ TeV. However, it could happen that anomalies in the WW scattering sector will be observed at such a machine, either because more complicated Higgs representations occur and/or because the WW scattering sector becomes strongly interacting at high energy. In this case, e^-e^- collisions are almost certain to be required in order to obtain a comprehensive picture of the nature of electroweak symmetry breaking. Thus, if at all possible, an option for e^-e^- collisions at the next linear collider, and especially its second-generation successor or extension, should be retained.

5. Acknowledgements

REFERENCES

1. See, for instance, J.F. Gunion, *Higgs Bosons at an e^+e^- Linear Collider: Theory and Phenomenology*, these proceedings.

2. For a review see J.F. Gunion, H.E. Haber, G.L. Kane, and S. Dawson, *The Higgs Hunter's Guide*, Addison-Wesley, Redwood City, CA (1990).

3. J.F. Gunion, R. Vega, and J. Wudka, *Phys. Rev.* **D43** (1991) 2322.

4. J.F. Gunion, R. Vega, and J. Wudka, *Phys. Rev.* **D42** (1990) 1673. See also, P. Bamert and Z. Kunszt, *Phys. Lett.* **B306** (1993) 335.

5. J.F. Gunion, H.E. Haber, and J. Wudka, *Phys. Rev.* **D43** (1991) 904.